STOCHASTIC MODELING ERROR REDUCTION USING BAYESIAN APPROACH COUPLED WITH AN ADAPTIVE KRIGING BASED MODEL

Ahmed Abou-Elyazied ABDALLH and Luc DUPRÉ

Department of Electrical Energy, Systems and Automation, Ghent University, Belgium

E-mail: Ahmed.Abdallh@ugent.be

Abstract. Magnetic material properties of an electromagnetic device can be recovered by solving an inverse problem where measurements are adequately interpreted by a mathematical forward model. The accuracy of the material properties recovered by the inverse problem is highly dependent on the accuracy of these forward models. In order to ensure the highest possible accuracy of the inverse problem solution, all physics of the electromagnetic device need to be perfectly modeled using for example a complex numerical model. However, the more accurate 'fine' models demand a high computational time and memory storage. Alternatively, less accurate 'coarse' models can be used with a demerit of the high expected recovery errors. Therefore, the Bayesian approximation error approach has been used for reducing the modeling error originating from using a coarse model instead of a fine model in the inverse problem procedure. However, the Bayesian approximation error approach may fail to compensate the modeling error completely when the used model in the inverse problem is too coarse. Therefore, there is a definitely need to use a quite accurate coarse model. In this paper, the electromagnetic device is simulated using an adaptive Kriging based model. The accuracy of this 'coarse' model is *a priori* assessed using the cross-validation technique. Moreover, the Bayesian approximation error approach is utilized for improving the inverse problem results by compensating the modeling errors. The proposed methodology is validated on both purely numerical and real experimental results. The results show a significant reduction in the recovery error within an acceptable computational time.

Keywords: Bayesian approach, inverse problem, Kriging models, modeling error.

INTRODUCTION

Recently, the magnetic parameters of the magnetic core material inside an electromagnetic device (EMD), such as rotating electrical machines, have been retrieved using a coupled experimental-numerical electromagnetic inverse problem [1]. In these inverse problems, the measurements are interpreted using a forward model where the difference between the numerical model responses and the measurement quantities is iteratively minimized using a minimization algorithm. In practice, two major aspects can reduce the accuracy of the recovered solution of the inverse problem, specifically: measurement noise and inaccurate modeling. Measurement noise can be reduced to some extent by accurately performing the measurements. On the other hand, modeling errors basically originate from two main sources: the uncertain 'geometrical' model parameters and the way of modeling the physical phenomena of the EMD. The effect of the uncertain geometrical model parameters on the solution of the inverse problem has been extensively investigated by the authors, see [1]. In this reference, the EMD models are assumed to be perfect, i.e. all physical phenomena are modeled, or in other words, the EMD models exactly simulate the reality. To this end, the EMD needs to be modeled using a very complex numerical model, e.g. 3D finite element (FE) model with a very fine mesh discretization level and including very detailed material models. However, these 'fine' forward models are computationally expensive, and consequently the computational time of the inverse problem becomes incredible due to its iterative nature. Alternatively, simplified fast but less accurate 'coarse' models can be used instead of the fine models. In this case, a recovery error in the inverse problem solution is expected due to the modeling error originating from the simplification in the used forward model. In order to compensate this modeling error, the Bayesian approximation error approach can be used by modifying the objective function to be minimized with the misfit between fine and coarse forward model responses. However, there are some drawbacks in this technique such as the considerable computational time and the assumption of having a stochastic modeling error with a uniform distribution. In this paper, we present a robust methodology in order to overcome these drawbacks.

METHODOLOGY

The proposed methodology comprises two steps: the first step is the simplification of the fine model using an adaptive Kriging based model, and the second step is the implementation of the Bayesian approximation error approach for compensating the modeling error.

An adaptive Kriging based model: Kriging models are widely used for modeling EMDs [2]. Suppose that the aim is to simplify the fine forward model: $y(\mathbf{u}) = f_f(\mathbf{u})$ using the Kriging model, where y is the forward model response and $\mathbf{u}$ is the vector of the unknown parameters that needs to be identified using the inverse problem. The most common form of Kriging models is $\hat{y}(\mathbf{u}) = \beta + E(\mathbf{u})$, where $\hat{y}(\mathbf{u})$ is the simplified response of the forward model, and β is a constant. $E(\mathbf{u})$ is the vector of stochastic modeling error with zero mean and variance σ^2. In fact, it is crucial to assess the accuracy of the simplified model before using it in the inverse procedure. To this end, we use the cross-validation scheme which is a very fast model accuracy assessment technique [2]. Based on the calculated accuracy level, the Kriging model is adapted iteratively until satisfying a predefined value of the accuracy level.

The Bayesian approximation error approach: Although the simplified model is considered to be accurate according to the model accuracy assessment technique, it still contains a modeling error. In order to reduce this modeling error, a Bayesian approximation error approach is utilized by adapting *a priori* the objective function to be minimized by the modeling error of the simplified Kriging model. If the objective function is $OF_{Trad}(\mathbf{u}) = \|y(\mathbf{u}) - w\|^2$ with w being the measured quantity corresponding to the modeled one, then the modified objective function is $OF_{Bayesian}(\mathbf{u}) = \|L_m(\hat{y}(\mathbf{u}) - w)\|^2$, with L_m being the reciprocal of the covariance of the modeling error.

The novelty of the presented approach over the one presented in [1] is three-fold. In [1], the Bayesian technique is applied to reduce the modeling error in the inverse problem solution when a 2D FE or an analytical model is used instead of a 3D FE model. These coarse models are computationally expensive compared to the Kriging coarse model. Moreover, the accuracy of the coarse model was fixed and not adapted; however, the accuracy of the Kriging coarse model used in this paper is improved to ensure an acceptable accuracy level. Furthermore, the application of the Bayesian approach is explicitly derived for modeling error with a normal distribution. In [1], the objective function is adapted by the *misfit* between the fine and the coarse model responses, which assumed to follow the normal distribution. This assumption is rarely occurred. The modeling error may or may not have a normal distribution. In fact, the distribution of the modeling error should be tested before applying the Bayesian approximation error approach. If the error can be approximated by the normal distribution, then the Bayesian approximation error approach can be used accurately. Nevertheless, a modeling error still appears. However, the modeling error due to the simplification of the fine model using the Kriging model has a stochastic nature and purely follows a normal distribution.

RESULTS AND DISCUSSION

In this digest, the proposed methodology is applied for a simple mathematical test function with only one unknown parameter: $f(x) = \sin x + \sin(10/3)x + \ln x - 0.84x, 2.7 \le x \le 7.5$. The Kriging model is built using N data of the fine model, and the root mean square error (*RMSE*) is calculated using the cross-validation technique. The number of data N is iteratively increased till the value of the RMSE becomes lower than a predefined value, e.g. 1%. Figs.1(a) and (b) show the responses of the Kriging model compared to the fine model response for two sets of data, i.e. N = 16 and 40, with 3.9% and 0.82% RMSE values, respectively. Fig.1(c) shows the recovery error with and without compensating the modeling errors for the two sets of data. It is clear from this figure that the modeling error is successfully reduced using the Bayesian approach for N = 40.

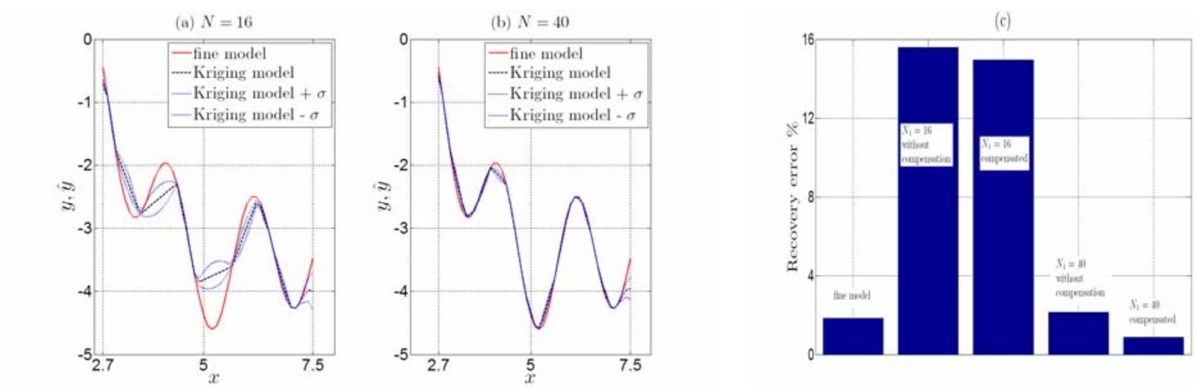

Fig. 1: (a) The behavior of the Kriging model for (a) N = 16, (b) N = 40 compared to the fine model. (c) The values of the recovery error of each mode with and without applying the Bayesian approach.

REFERENCES

[1] A. Abdallh and L. Dupré. Magnetic material characterization using an inverse problem approach, book chapter in Advanced Magnetic Materials, ISBN 979-953-307-547-0, InTech, 2012.

[2] L. Wang and D. Lowther. Selection of approximation models for electromagnetic device optimization. IEEE Transactions on Magnetics, vol. 42, no. 4, pp. 1227-1230.